

# ▲ PROVINCIAL DIGITAL LEARNING RESOURCES – WHAT'S NEW? FRACTION STRIPS TOOL



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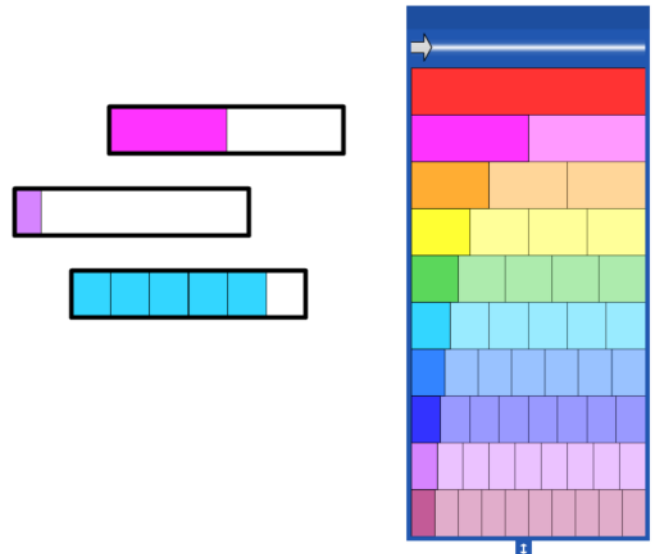
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The most recent addition to the mathies collection ([www.mathies.ca](http://www.mathies.ca)) of digital learning tools is an updated version of Fraction Strips Tool, which is now available for mobile devices as well as desktop computers. The revised version includes the often-requested annotation tools, image import, and undo/redo functionality. In addition, this version of the tool includes an infinite workspace that is pannable/scrollable and the ability to Zoom In, Zoom Out, and Zoom to Fit.

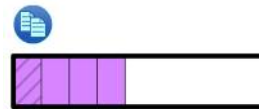
Let's explore the functionality of the Fraction Strips Tool by doing some math!

## Representing Fractions

The tower contains fraction strips that are equi-partitioned into pieces. As students reason about the part-whole relationship represented by each strip in the tower, they count the number of pieces that make up a whole and name them as fractions; e.g., there are five equal-sized green pieces that make up a whole, and so one green piece represents  $\frac{1}{5}$ .



Drag fraction pieces from the tower into the workspace to create representations of various fractions. Drag the first piece on the left of a strip, to create a unit fraction. Starting with a piece further to the right in the tower creates other proper fractions. For example, drag the fifth piece in the row that is partitioned into sixths to create five-sixths. Pieces are always placed within a whole so that the part-whole relationship is reinforced.




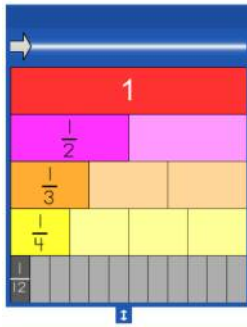
When fraction pieces are selected, a copy button appears. Students could practise their unit fraction counting skills by selecting a unit fraction, like one-ninth, and pressing its copy button repeatedly, counting 2 one-ninths, 3 one-ninths, etc.


The tool can also be used to create mixed fractions, improper fractions and fractions made up of unlike pieces.



As students represent various fractions, they may wish to adjust some of the tool options to assist them in their work.

Use the Customize Tower button  to choose a colour palette (Rainbow, Original, or One Colour) or to change the colour of all the fractions pieces of a specified size. It is also possible to include or exclude strips from the tower. For example, you might want to set up a tower that shows only a specific set of strips, as illustrated here.





Additionally, students can control whether or not labels are showing. Press the label button  to toggle between showing: no labels, the labels in the tower only, or labels in both the tower and workspace.

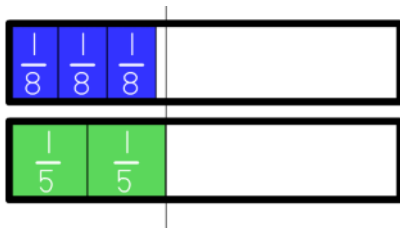
### Suggested Representation Activities:


- 1) Represent various fractions in more than one way.
- 2) Represent a fraction that is really close to 1. How do you know?

### Comparing and Ordering Fractions

The Fraction Strips Tool has been designed to facilitate the comparison of fractions. While fraction strips are technically an area model, students are able to compare fractions by attending only to the length, since the height is constant. A key feature of the tool is that the size of the whole is consistent for all representations. Consider that one-half of a large pizza and one-half of a small pizza are quite different quantities, since the wholes are different. Also, fraction strips can easily be aligned using the built-in snapping feature, which ensures that two fraction strips have the same starting position.

Students can use the Zoom In  button to examine representations with greater precision. They might also activate the Vertical Comparison Bar  to help visually track from one fraction strip to the other.



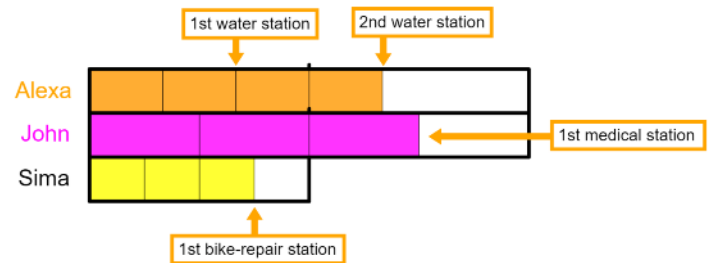
After zooming in, press the Zoom to Fit  button to re-orient the workspace so that all fraction pieces are once again visible on the screen.

Once students have compared fractions, they can arrange them in the workspace to order them, using whatever rules they would like.

### Suggested Comparison Activities:


- 1) Name two fractions. Which do you think is larger? Why? Use the tool to verify.
- 2) Solving problems like the *Bike-a-thon Task*:  
In a bike-a-thon, cyclists will find:
  - water stations every two-thirds of a kilometre
  - medical stations every three-halves of a kilometre
  - bike-repair stations every three-fourths of a kilometre
 John has reached the first medical station, Alexa is at the second water station, and Sima is at the first bike-repair station.

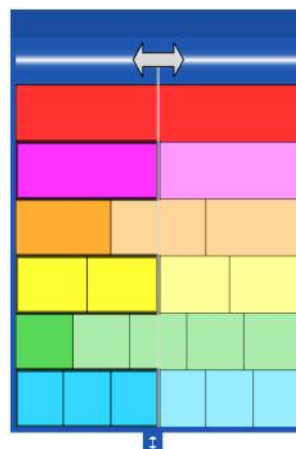
Who is furthest along the course? (Adapted from Nelson Mathematics 6, 2006)




A detailed analysis of one solution of the bike-a-thon problem, including mathematical and learning needs considerations, is available on the support wiki page for the Fraction Strips Tool ([www.mathclips.wikispaces.com/FractionStripsTool#PDF](http://www.mathclips.wikispaces.com/FractionStripsTool#PDF)).

### Equivalent Fractions

Understanding the concept of equivalent fractions is one of the fundamental building blocks needed for deep fraction understanding. Drag the Equivalency Bar , located at the top of the Fraction Strip Tower, to explore various equivalencies. Equivalent fractions are highlighted as the bar moves across the tower. This image shows that  $\frac{1}{2}$ ,  $\frac{2}{4}$ , and  $\frac{3}{6}$  are equivalent.

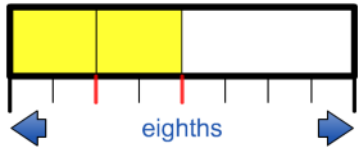


In the workspace, use the Rulers  button to explore various equivalencies. The Fraction Strips Tool helps students understand two important equivalent fraction concepts.

#### 1) Splitting:

This image shows that  $\frac{2}{4}$  is equivalent to  $\frac{4}{8}$ . Here, 2 one-fourth pieces have been dragged to the workspace, the rulers have been set to show

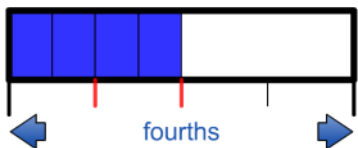
the number stepper, and the stepper has been changed from fourths to eighths. Two ticks have turned red to indicate that the end of the piece lines up exactly, allowing a student to conclude that  $\frac{2}{4}$  is equivalent to  $\frac{4}{8}$ . Students can continue to change the stepper. Notice which fractional units allow 2 one-fourths to be named nicely and which do not.



In fact, the representation supports students in understanding why multiplying both the numerator and denominator of a fraction by the same value determines an equivalent fraction. In this case, the eighth ruler shows the partition, or split, of each one-fourth piece into two equal pieces. This effectively doubles the number of pieces that are shaded yellow (the numerator), as well as the number of parts in the entire whole (the denominator).

**2) Merging:**

Another way to see that  $\frac{2}{4}$  is equivalent to  $\frac{4}{8}$  is to start by representing 4 one-eighth pieces. Next, adjust the steppers while watching for the red tick marks to indicate an exact match. The representation illustrated shows that every 2 one-eighth pieces can be merged to form a one-fourth piece. This is what happens when we form equivalent fractions by dividing both the numerator and denominator by the same value. Here we divide by two, which halves both the number of shaded pieces (the numerator) and the number of parts in the entire whole (the denominator). Students can continue to adjust the stepper to find other equivalent fractions that are created by merging smaller parts to make larger parts.



**Fraction Operations**

The Fraction Strips Tool can also be used to model the various operations and help students to develop conceptual understanding of these operations.

We will consider each of the four operations, using the fractions  $\frac{4}{5}$  and  $\frac{1}{4}$ .



**Addition of Fractions**

Let's start with the sum of  $\frac{4}{5}$  and  $\frac{1}{4}$ .

Before asking students to actually determine this sum, ask them to decide whether the result will be less than, greater than, or equal to 1. If we ask students to use their reasoning and proving skills to estimate the value of an answer prior to calculating it, they are more likely to reflect on the reasonableness of their answer after employing a standard algorithm.

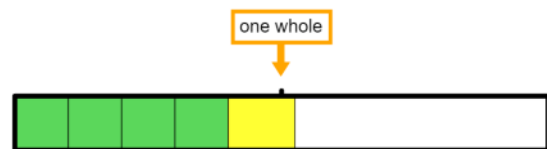
In this case, students can use the tool to reason that the sum of  $\frac{4}{5}$  and  $\frac{1}{4}$  will be greater than 1. Their reasoning might go something like this:

$\frac{4}{5}$  is one-fifth away from 1.


$\frac{1}{4}$  is larger than  $\frac{1}{5}$  (since a whole partitioned into fourths has fewer pieces than the same whole partitioned into fifths, so the fourth pieces must be larger than the fifth pieces). *Notice in the representation that the yellow one-fourth piece is definitely bigger than the green one-fifth piece.*

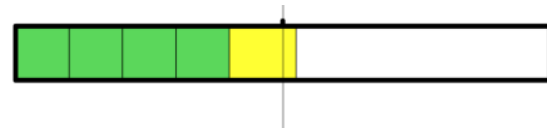
So adding a piece that is larger than  $\frac{1}{5}$  to  $\frac{4}{5}$  will result in a fraction that is larger than 1.

The operation of addition can be modelled by dragging the  $\frac{1}{5}$  piece into the same whole as the  $\frac{4}{5}$  piece to combine the two fractions.

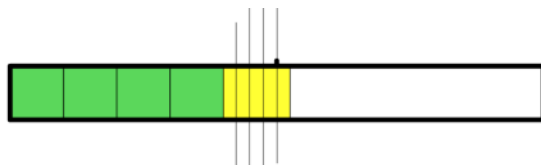


The visual representation quickly confirms the prediction that  $\frac{4}{5} + \frac{1}{4}$  is larger than 1. To find the precise sum, we need to determine how much larger it is.


Using a Vertical Comparison Bar  to indicate the whole can help with reasoning about the size of the piece that is to the right of one whole.

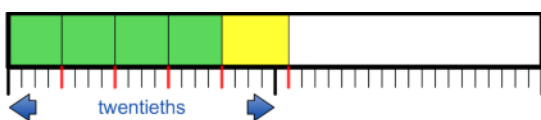


Use additional vertical comparison bars to equally partition the yellow one-fourth piece.



Visually, it would seem that the piece we are trying to name is  $\frac{1}{5}$  of  $\frac{1}{4}$ . Here, we intentionally use non-specific language, since we are working with pretty small pieces and it is difficult to be certain if our partitioning is exact. If each of the 4 one-fourth pieces in a whole is partitioned into 5 equal pieces, there would be 20 pieces altogether. So, the sum of  $\frac{4}{5}$  and  $\frac{1}{4}$  looks like it could be  $1\frac{1}{20}$ .

Use the Ruler feature  of the Fraction Strips Tool to confirm this reasoning. Clicking the button once shows the ruler; clicking again shows the fractional unit stepper.



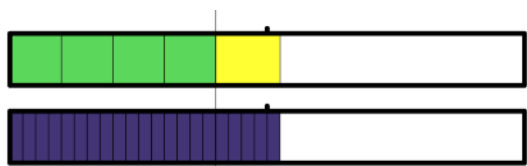
Adjust the stepper to twentieths. As the stepper is changed, notice that some ticks are black and some are red. Recall that the red ones indicate the piece matches that tick mark exactly. So, the red tick mark at the end of the yellow piece indicates that this strip is exactly  $1\frac{1}{20}$ .

Students can also use this representation to see that:

- $\frac{1}{5} = \frac{4}{20}$  (each green piece is split into 4 equal parts),
- $\frac{4}{5} = \frac{16}{20}$  (4 green pieces, each split into 4 equal parts gives 16 pieces), and
- $\frac{1}{4} = \frac{5}{20}$  (the yellow piece is split into 5 equal parts).

Hence, this representation might lead students to discover the standard algorithm for adding fractions, finding a common denominator.  $\frac{4}{5} + \frac{1}{4} = \frac{16}{20} + \frac{5}{20} = \frac{21}{20}$

The model can also be used to show converting to twentieths.



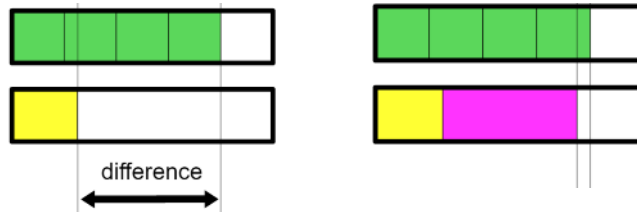
## Subtraction of Fractions

First let's consider subtraction as the difference between  $\frac{4}{5}$  and  $\frac{1}{4}$ .

Once again, before computing, encourage students to estimate an answer.

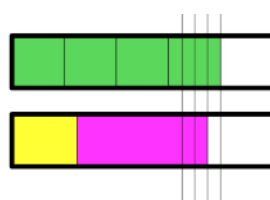
The Fraction Strips Tool visual representation shows us

that the difference is a bit more than  $2\frac{1}{2}$  fifths, so the difference is just a bit larger than  $\frac{1}{2}$  of the whole.



Confirm this reasoning by dragging a one-half piece into the strip beside the yellow one-fourth piece, to see that there is an additional white piece needed to make up the difference.

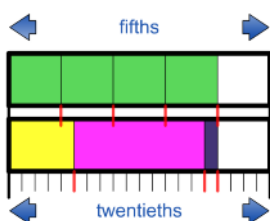
Students can reason about the size of this piece by creating additional vertical bars and using them to equally partition the one-fifth piece, much as we did for addition.



It seems that the missing piece we are looking for is one-fourth of one-fifth, which is one-twentieth (5 pieces per whole, each cut into 4 equal pieces).

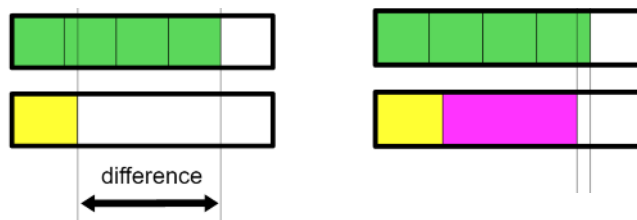
So the difference appears to be  $\frac{1}{2}$  plus  $\frac{1}{20}$ . Using their previously developed understanding of equivalent fractions, students can rename  $\frac{1}{2}$  as  $\frac{10}{20}$ . An additional  $\frac{1}{20}$  gives a total difference of  $\frac{11}{20}$ .

As with addition, turn on the rulers to confirm this reasoning and think about the common-denominator algorithm.




Alternatively, to model subtraction as the act of removing or taking something away, start by creating an equivalent representation of  $\frac{4}{5}$  that includes a  $\frac{1}{4}$  piece. Reasoning similar to the

difference thinking above can be used to model this representation. Then, simply remove the yellow  $\frac{1}{4}$  piece.



## File Operations

One of the exciting new features of mathies tools is the ability to open and save files. Once you have used the

Fraction Strips Tool to solve a problem, open up the Settings  and press Save.



Pressing the Save button will bring up the system dialog with a suggested file name and file type (e.g., .FractStrips for Fraction Strips). It is recommended that you do not change the


file type, since the *Open* operation only finds files with that extension (or with .txt and .xml). The suggested file name is based on the date and time of saving and should be unique.

To retrieve this saved file at a later time, use the Open File button in Settings. Pressing this button will bring up the system dialog that allows you to find the folder in which you have saved the file.

Open and Save functionality is also available on mobile devices. Files saved on one device can be opened on another! For detailed support, you can visit [www.mathclips.wikispaces.com/File+Operations](http://www.mathclips.wikispaces.com/File+Operations).

Currently, Fraction Strips saves and opens only the actual state of the tool at the time of saving. When you open a file, the representations are entirely interactive and you can continue to add to your work. In the near future, you will also be able to undo back through the history of steps that led up to the point of saving. The development team believes that this feature will support a variety of assessment opportunities for students to explain their thinking.

## Feedback and Future Requests

Please feel free to send us your feedback about any mathies tool, using the Feedback Form button inside the Information Dialog, accessed from the  button. Visit the support wiki page for more examples and detailed descriptions of the functionality of the tool.



You can also send your comments to **WhatsNew@oame.on.ca**.

You can share your experiences on Twitter, using the hashtag **#ONmathies**, and follow or message us at **@ONmathies**. There is an increasing set of interesting posts of student and teacher work on Twitter. To be among the first to find out about the latest digital-tool developments, sign up for our email list at [www.mathclips.ca/WhatsNewEmailList.html](http://www.mathclips.ca/WhatsNewEmailList.html). ▲

