▲ Provincial Digital Learning Resources – What's New? Exploration of Fraction Operations




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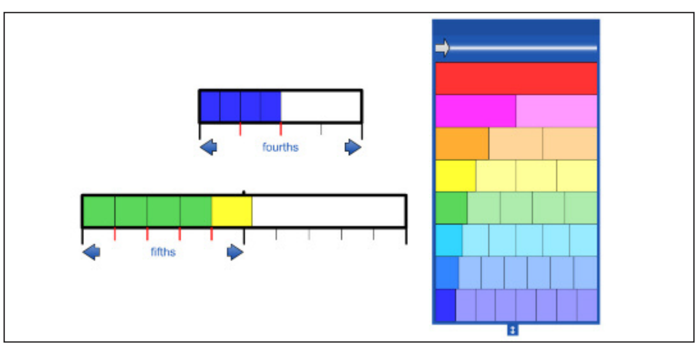
In the September 2017 issue of the *Gazette*, some of the features of the Mathies Fraction Strips tool were explored. The tool was used to represent, compare, and order fractions, as well as think about different ways to visualize equivalent fractions. The exploration of fraction operations focused on how the tool might help students develop an understanding of what it means to add and subtract fractions. In this issue, the exploration of fraction operations will continue by considering how powerful visual representations can support the understanding of multiplication and division.



Multiplication of Fractions

Students often have the misconception, based on their experience with whole numbers, that multiplying always results in a larger quantity. Modelling questions, using the Fraction Strips Tool, will help students understand that this is not always the case.

Students should be encouraged to estimate an answer before calculating the result. Unfortunately, students sometimes fail to appreciate the value of estimation. Perhaps this is because they see it simply as an exercise in rounding. Rather, students should be encouraged to think about what size of an answer would be reasonable, given the values and operation in the question. Some questions to consider might be:

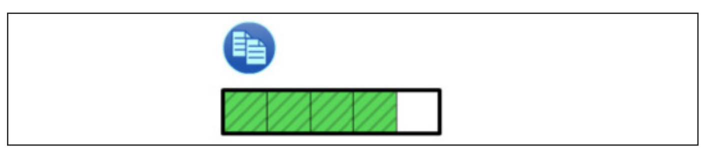
- How do I expect my answer to compare to various benchmarks? (e.g., one-half, one whole)
- How do I expect my answer to compare to the values in the question? Why?
- What answer would be much too large?
- What answer would be much too small?

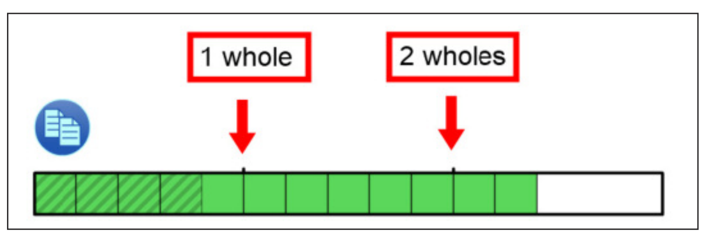
Students who are confident in their ability to use their number sense skills to predict an answer prior to calculating are more likely to look for errors if their calculated answer does not match their prediction.

Multiplying a Whole Number and a Fraction

Multiplying a fraction by a whole number can be thought of as repeated addition and modelled by making the appropriate number of copies of the fraction. For example, $3\times\frac{4}{5}$ can be thought of as 3 groups of $\frac{4}{5}$. Students might estimate that this product will be somewhere between 2 and 3, since $\frac{4}{5}$ is a bit smaller than 1.

Represent $\frac{4}{5}$, then press the copy button twice to model a total of three $\frac{4}{5}$.



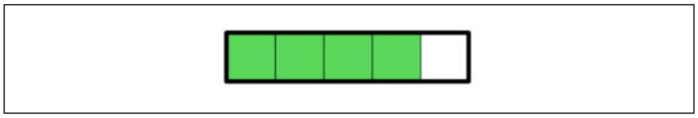


Now, name the result. So this product is either 12/5 (count the number of green one-fifth pieces: 1 one-fifth, 2 one-fifths, 3 one-fifths, etc.) or $2\frac{2}{5}$ (which is visually supported by the black tick marks at the end of each whole).

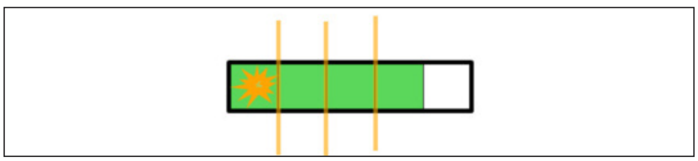
Multiplying Proper (or Improper) Fractions

One way to calculate $\frac{1}{4}\times\frac{4}{5}$ is to think about this expression as $\frac{1}{4}$ of $\frac{4}{5}$. When thinking about estimating products of proper fractions, students will come to understand that, in this case, multiplication actually results in a smaller result, since you are finding a part of some other value.

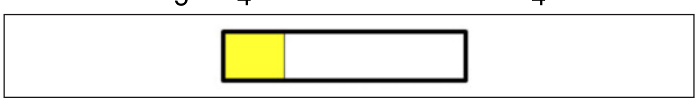
Start by representing $\frac{4}{5}$.



To determine one-fourth of something, you need to partition the quantity into 4 equal parts. In this case, that is straightforward, since the fraction four-fifths is made up of 4 one-fifth pieces. So $\frac{1}{4}$ of $\frac{4}{5}$ is $\frac{1}{5}$, which is 1 green piece.

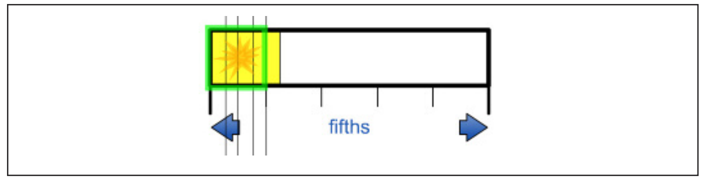


Multiplication is commutative, so $\frac{4}{5}\times\frac{1}{4}$ gives the same result, but modelling this requires us to think differently as we determine $\frac{4}{5}$ of $\frac{1}{4}$. Start by representing $\frac{1}{4}$.



To determine $\frac{4}{5}$ of this quantity, we need to partition it into 5 equal parts and then select 4 of them. The Vertical Comparison Bar, together with our spatial reasoning skills, allows us to create this partition.

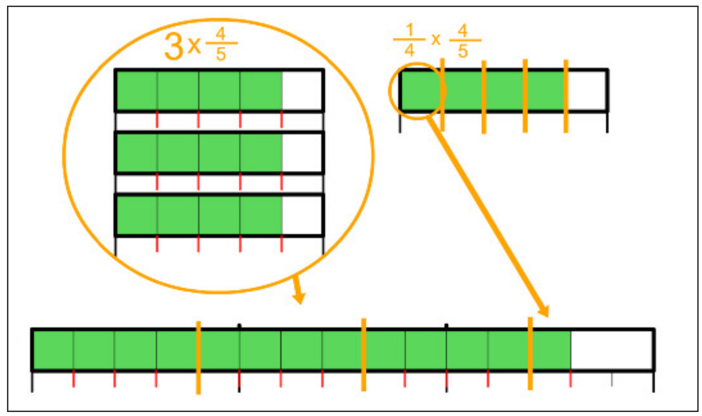
How big is each of these pieces? There are 4 one-fourths in the whole (black rectangle). If each piece is partitioned into 5 equal pieces, then there will be 20 equal pieces in 1 whole. So, each piece is 1/20. We need 4 of these pieces, so we have 4/20, which simplifies to $\frac{1}{5}$ as expected.



Modelling the multiplication of various pairs of fractions can lead students to better understand why the standard algorithm works.

Multiplying Mixed Numbers and Fractions

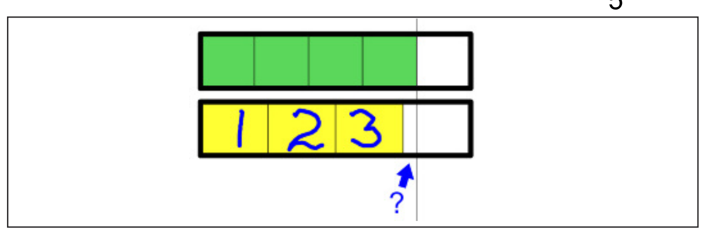
Combining these ideas allows us to model products such as $3\frac{1}{4}\times\frac{4}{5}$ by using the distributive property to break this product into partial products. First determine $3\times\frac{4}{5}$, then determine $\frac{1}{4}\times\frac{4}{5}$, and add these two results together.



We commonly think of $\frac{4}{5}$ of a quantity as the result of splitting the quantity into 5 equal pieces and combining 4 of them. However, we can also think of $\frac{4}{5}$ as 4 wholes shared equally among 5. Here, then, we could have created 4 copies of $3\frac{1}{4}$ and taken $\frac{1}{5}$ of the result. Four copies of $3\frac{1}{4}$ combine to make 13, and $\frac{1}{5}$ of 13 is $\frac{13}{5}$, as above.

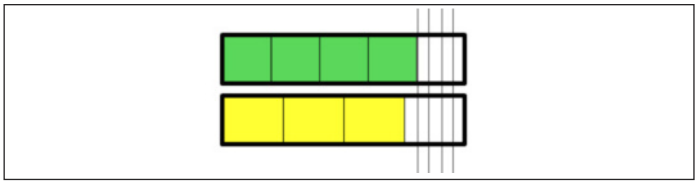
Division of Fractions

One way to calculate $\frac{4}{5}\div\frac{1}{4}$ is to start by representing $\frac{4}{5}$, then think about how many one-fourth pieces fit in $\frac{4}{5}$.

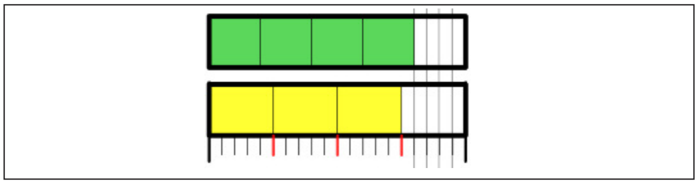


Using the vertical comparison bar to draw a line at the end of the $\frac{4}{5}$ green strip helps us to see that the answer is 3 and a bit. What fraction of a one-fourth piece is this bit?

We employ the equal partitioning idea described above to figure this out. Copy additional comparison bars and place them so they partition the white space at the end of the yellow strip into roughly equal pieces.

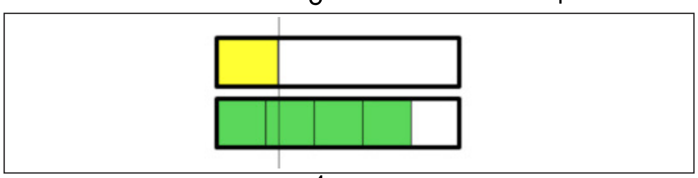


It looks like the extra bit is $\frac{1}{5}$ of a one-fourth piece, so the quotient may be $3\frac{1}{5}$. This visual representation helps students to give meaning to the numeric answer. There are 3 and $\frac{1}{5}$ yellow one-fourth pieces in $\frac{4}{5}$. Turning the rulers on helps ensure that we have partitioned the one-fourth piece accurately.



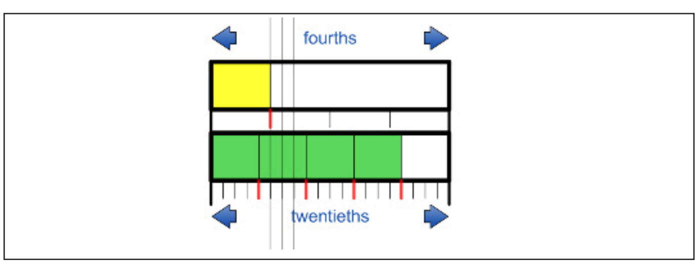
As teachers, we know that division is not commutative, so we would expect $\frac{1}{4} \div \frac{4}{5}$ to give an entirely different result. It would be interesting to see whether or not students are surprised by the different answer.

It is helpful to think about this division slightly differently, since $\frac{4}{5}$ is larger than $\frac{1}{4}$. This means that only some portion of the $\frac{4}{5}$ piece will fit inside $\frac{1}{4}$. So, the question we need to explore is how much of a $\frac{4}{5}$ piece fits inside a $\frac{1}{4}$ piece?



Only a portion of the $\frac{4}{5}$ piece will fit, so we know the answer will be less than 1. In fact, visually, it looks like the answer will be just a bit larger than $\frac{1}{4}$ of the $\frac{4}{5}$ strip. Again, the question is, how big is this little bit? That is, what part of a one-fifth piece is this extra bit?

Equally partition the one-fifth piece. Use the rulers to convince yourself that you have done this accurately.



The extra bit looks like it is one-twentieth of the whole or, more importantly, for our purposes, one-fourth of a one-fifth piece. This means that there would be 16 such pieces in the green four-fifths strip altogether. So, the yellow one-fourth piece is made up of $\frac{5}{16}$ of the green four-fifths strip, four-sixteenths from the first piece, plus one additional sixteenth; that is, $\frac{5}{16}$ of the green $\frac{4}{5}$ strip fits in $\frac{1}{4}$.

Division is definitely not commutative, but comparing the two quotients is interesting: What is the relationship between $3\frac{1}{5}$ and $\frac{5}{16}$? What do you notice when you write the first as an improper fraction? What is the product of these two fractions?

Have you ever thought of using common denominators to divide? Let's revisit the same two division questions, using this strategy. See Figure 1.

The question of how many of one quantity fit into another is the standard "goes into" division situation. We should not be surprised in the first column that the answer is 16 ÷ 5, or 16/5, and in the second column, that the answer is 5 ÷ 16, or 5/16. Thinking of a fraction as a quotient, the answer to a division question is an important way we use fractions.

Other Fraction Supports

Teaching and learning about fractions in a meaningful way can certainly be challenging! Here are some other Ontario Ministry of Education resources that support the effective development of fraction concepts.

Gap-Closing ePractice Activities

Visit **www.mathies.ca/additionalSupports.html** to access ePractice resources. Topics include representing, comparing, and operating with fractions. These digital interactive activities are designed to provide practice opportunities for students and support conceptual understanding by providing detailed feedback. The operation activities illustrate two different ways of thinking about each operation.

Fractions Learning Pathways

A research informed framework found at **www.edugains.ca/newsite/DigitalPapers/FractionsLearningPathway/index.html** is a helpful planning tool for teaching fractions. It includes a range of field-tested tasks that have proven to be effective in the teaching and learning of fractions in Ontario schools.

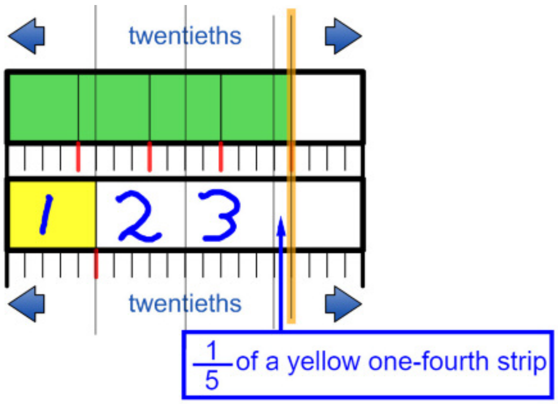
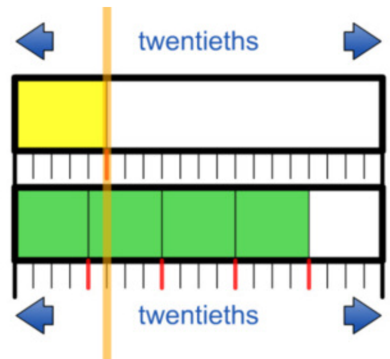
<table>
<tr><th></th><th>$\frac{4}{5} \div \frac{1}{4}$</th><th>$\frac{1}{4} \div \frac{4}{5}$</th></tr>
<tr><td>Step 1:</td><td colspan="2">Represent both one-fourth and four-fifths.</td></tr>
<tr><td>Step 2:</td><td colspan="2">Rename these fractions, using the same fractional unit (in this case, twentieths).
As illustrated in the diagram,
• $\frac{1}{4}$ is equivalent to $\frac{5}{20}$
• and $\frac{4}{5}$ is equivalent to $\frac{16}{20}$.</td></tr>
<tr><td></td><td>
</td><td>
</td></tr>
<tr><td>Step 3:</td><td>Consider: “How many five-twentieths fit in sixteen-twentieths?”</td><td>Consider: “How many of the sixteen-twentieths fit in the five-twentieths?”</td></tr>
<tr><td>Step 4:</td><td>The diagram shows that 3 yellow pieces plus $\frac{1}{5}$ of a yellow piece will fit, so the answer is $3\frac{1}{5}$ yellow one-fourth pieces.</td><td>In fact, only 5 out of 16 of them fit, so the answer is $\frac{5}{16}$ of the green four-fifths strip.</td></tr>
</table>

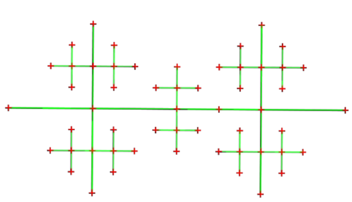
Figure 1

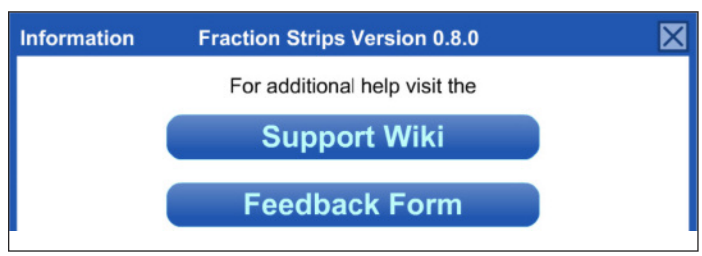
Ontario Fractions Educational Research

A growing body of research connected to the teaching and learning of fractions can be found at **www.edugains.ca/newsite/math/fractions.html**. Resources found here include the Paying Attention to Fractions document, as well as a collection of Research Summaries outlining key ideas for teachers to consider when planning and delivering lessons such as the use of Purposeful Representations and ideas about how to build understanding of Unit Fractions.

Feedback and Future Requests

Please feel free to send us your feedback about any Mathies tool, using the Feedback Form button inside the Information Dialog. Visit the support wiki page for more examples and detailed descriptions of the functionality of the tool.





You can also send your comments to **WhatsNew@oame.on.ca**. You can share your experiences on Twitter, using the hashtag #ONmathies, and follow or message us at @ONmathies. There is an increasing set of interesting posts of student and teacher work on Twitter.

To be among the first to find out about the latest digital tool developments, sign up for our email list at **www.mathclips.ca/WhatsNewEmailList.html**. Watch for announcements regarding exciting improvements to annotations, including new drawing objects (rulers, number lines, grids, hops, and polygons), as well as the ability to copy and rotate all annotation objects. These improvements will show up first as updates to existing tools, and then in the two new apps in development. Stay tuned! ▲